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CAPILLARY ADHESION IN THE CONTACT BETWEEN ELASTIC SOLIDS†

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The problem of the indentation of an axisymmetric punch, the shape of which is simulated by a power function, into an elastic half-space when there is a liquid forming a meniscus in the gap is considered. The results are used to analyse the dependence of the contact characteristics and the capillary adhesion forces on the amount of liquid in the meniscus, the value of the applied load and the punch shape. The range of applicability of the simplified approach in which elastic deformations of the half-space outside the contact area are ignored is estimated.

The presence of water vapour in the atmosphere leads to the formation of thin films of liquid on the surface of solids. When such surfaces interact, capillary effects play an important role. Thus, it has been shown experimentally [1, 2], that the adhesion force when a magnetic disc interacts with a head increases considerably with increasing humidity of the surrounding air, this can be a reason for the surface damage.

A formula $f_a = 4\pi R\sigma \cos\theta$ (where σ is the surface tension of the liquid and θ is the wetting angle) was obtained in [3] for the capillary adhesion force between a plane and rigid hemispherical asperity of radius R. According to this formula the adhesion force is greater for more mildly sloping asperities and is independent of the amount of liquid. When similar relations were used for the force of capillary adhesion between the asperities of interacting rough surfaces, three forms of contact were distinguished depending on the extent to which the gap was filled with liquid [4].

When calculating the force of capillary adhesion between rough surfaces the following formula was used for the force acting on an individual asperity [5]

$$f_a = 4\pi R\sigma(1+\delta) \tag{0.1}$$

where δ is a parameter which depends on the thickness of the liquid film and the asperity deformation. Here the elasticity of the asperities was taken into account, but it was assumed that the pressure of the liquid has no effect on any change in their shape. This approach, however, does not enable the effect of capillary forces on the stress–strain state of the contacting bodies to be estimated.

In this paper we solve the more rigorously formulated problem of capillary adhesion when a single asperity, simulated by punches of different shape, interacts with an elastic half-space.

1. FORMULATION OF THE PROBLEM

Consider the penetration, with a force q, of a rigid axisymmetric punch into an elastic half-space when there is a liquid present, which forms a meniscus in the gap between the contacting solids (Fig. 1). The punch surface is described by a smooth function $f(r) = Ar^{2n}$, where n is an integer.

A uniform pressure, which is less than atmospheric pressure by an amount

$$p_0 = \sigma(1/R_1 + 1/R_2)$$

acts in a ring-shaped region $a \leqslant r \leqslant b$ on the elastic half-space, where σ is the surface tension of the liquid, and R_1 and R_2 are the radii of curvature of the side surface of the meniscus. Assuming that the wetting angle is zero, and that the punch has a mildly sloping shape, i.e. $f'(b) \ll 1$, we can write

$$R_1 \approx h(b)/2,\quad R_2 \approx b;\quad h(r) = f(r) - f(a) + u(r) - u(a) \tag{1.1}$$

where $h(r)$ is the value of the gap and $u(r)$ are the normal displacements of the boundary of the elastic half-space. Assuming that $h(b) \ll b$, we obtain

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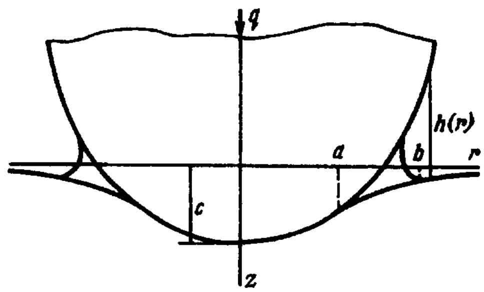

Fig. 1.

$$p_0 = 2\sigma/h(b) \tag{1.2}$$

It should be noted that the tension force of the liquid film $f_s = -2\pi b\sigma$ also acts on the elastic half-space round a circle $r = b$, which, in view of the above assumption, is directed along the tangent to the boundary of the elastic half-space. Simple estimates show that this force is much less than the force $f_L = -\pi(b^2 - a^2)p_0$ due to the Laplace pressure. In fact

$$\frac{f_s}{f_L} = \frac{bh(b)}{b^2 - a^2} \sim f'(b) \ll 1$$

Henceforth the force f_s will be neglected.

Assuming the atmosphere pressure to be zero, we obtain the following conditions on the boundary of the elastic half-space

$$\begin{aligned} &r \leqslant a: && u(r) = -f(r) + c \\ &a < r \leqslant b: && p(r) = -p_0 \\ &r > b: && p(r) = 0 \end{aligned} \tag{1.3}$$

where c is the punch penetration. In view of the smoothness of the punch, the following condition must also be satisfied on the boundary of the contact region

$$p(a) = -p_0 \tag{1.4}$$

The normal displacements $u(r)$ of the boundary of the elastic half-space due to the action of the normal pressures $p(r)$ are given by the well-known expression [6]

$$\begin{gathered} u(r) = A[p(r), b]; \quad 0 \leqslant r \leqslant b \\ A[p(r),\ x] = \frac{4}{p^*}\int_0^x p(r')K\left(\frac{2\sqrt{rr'}}{r+r'}\right)\frac{r'dr'}{r+r'}, \quad p^* = \frac{\pi E}{1-\nu^2} \end{gathered} \tag{1.5}$$

where $K(x)$ is the complete elliptic integral of the first kind.

We will assume that we are given the volume of the liquid υ in the meniscus, which is related to the geometry of the gap by the relation

$$\upsilon = 2\pi\int_a^b rh(r)dr \tag{1.6}$$

Finally, it follows from the equilibrium condition that

$$q = 2\pi\int_a^b rp(r)dr \tag{1.7}$$

Relations (1.2)–(1.7) enable us to determine the unknown functions $p(r)$ and $u(r)$ and the quantities a, and p_0.

2. METHOD OF SOLUTION

We will represent the function $p(r)$ in the interval $[0, a]$ in the form

$$p(r) = p_1(r) - p_0 \tag{2.1}$$

In this case the condition $p_1(a) = 0$ is satisfied at the point $r = a$. Then, taking (1.3) into account, for $0 \leqslant r \leqslant b$ we obtain from (1.5)

$$u(r) + 4P_0 bE(r/b) = A[p_1(r), a], \quad P_0 = p_0/p^* \tag{2.2}$$

When deriving (2.2) we used the value of the integral [7]

$$\int_0^b K\left(\frac{2\sqrt{rr'}}{r+r'}\right)\frac{r'dr'}{r+r'} = \begin{cases} bE(r/b), & r \leqslant b \\ r[E(b/r) - (1-(b/r)^2 K(b/r)], & r > b \end{cases} \tag{2.3}$$

where $E(x)$ is the complete elliptic integral of the second kind.

Taking (2.1) into account, expression (1.7) takes the form

$$q + \pi p_0 b^2 = 2\pi \int_0^a r p_1(r) dr \tag{2.4}$$

On the basis of condition (1.3), when $r \leqslant a$, expression (2.2) can be represented in the form of the integral equation

$$A[p_1(r), a] = -f_1(r) + c \tag{2.5}$$

to determine the pressures $p_1(r)$ under the punch, the shape of which is described by the smooth function

$$f_1(r) = f(r) - 4P_0 bE(r/b) \tag{2.6}$$

When $a < r \leqslant b$ the right-hand side of (2.2) then defines the displacements of the boundary of the elastic half-space outside the contact area with the punch, while the right-hand side of (2.4) corresponds to the force applied to this punch.

We will use the solution of the problem of the indentation of an axisymmetric punch of given shape $f_1(r)$ into an elastic half-space, obtained previously in [8], on the basis of which we obtain, from relations (2.1)–(2.5), expressions for the normal pressures and displacements at the boundary of the elastic half-space

$$p(r) = \pi^2 a^2 p^* \int_0^1 \frac{y}{\sqrt{1-y^2}} \int_{r/a}^1 \frac{x}{\sqrt{x^2a^2 - r^2}} \Delta f_1(axy) dx dy - p_0, \quad r \leqslant a \tag{2.7}$$

$$u(r) = \frac{2a}{\pi}\int_0^1 \frac{y}{\sqrt{1-y^2}} \int_0^1 \frac{x}{\sqrt{r^2 - x^2a^2}} \frac{\partial}{\partial x}(xc - xf_1(axy)) dx dy - 4P_0 bE\left(\frac{r}{b}\right), \quad a < r \leqslant b \tag{2.8}$$

and also the condition for determining the penetration of the punch

$$\int_0^1 \frac{y}{\sqrt{1-y^2}} \frac{\partial}{\partial y}(cy - cf_1(ay)) dy = 0 \tag{2.9}$$

Using Galin's formula [6] for the force acting on an axisymmetric punch and taking expression (2.4) into account, we obtain

$$q = p^*\left[\frac{2}{\pi}\int_0^a \Delta f_1(r) r\sqrt{a^2 - r^2} dr - P_0 b^2\right] \tag{2.10}$$

Expanding the elliptic integral in series and using the fact that $f(r) = Ar^{2n}$, we can convert expression (2.6) to the form

$$f_1(r) = Ar^{2n} + 2\pi P_0 b\left(\sum_{m=1}^{\infty}\left[\frac{(2m-1)!!}{(2m)!!}\right]^2 \frac{1}{2m-1}\left(\frac{r}{b}\right)^{2m} - 1\right) \tag{2.11}$$

To determine the required expressions for the pressure $p(r)$ and the displacements $u(r)$, and also the values of the penetration c and the load q, we substitute (2.11) into (2.7)–(2.10) and introduce the dimensionless quantities

$$\rho=\frac{r}{a},\quad P=\frac{p}{p^*},\quad Q=A^{2/(2n-1)}\frac{q}{p^*},\quad K=\frac{\sigma}{p^*}$$

$$U=A^{1/(2n-1)}u,\quad V=A^{3/(2n-1)}v,\quad C=A^{1/(2n-1)}c$$

$$\alpha=A^{1/(2n-1)}a,\quad \beta=A^{1/(2n-1)}b,\quad \gamma=\frac{a}{b}$$

We finally obtain

$$P(\rho)=\frac{(\beta\gamma)^{2n-1}}{\pi^2}\left[\frac{(2n)!!}{(2n-1)!!}\right]^2\sqrt{1-\rho^2}\sum_{k=1}^{n}\frac{(2k-3)!!}{(2k-2)!!}\rho^{2(n-k)}-$$
$$-P_0\left(1-\frac{2}{\pi}\operatorname{arctg}\rho\sqrt{\frac{1-\rho^2}{1-\gamma^2}}\right),\quad \rho\le 1 \tag{2.12}$$

$$U(\rho)=\frac{2}{\pi}(C-(\rho\beta\gamma)^{2n})\arcsin\frac{1}{\rho}+\frac{2(\beta\rho)^{2n}}{\pi}\sqrt{\rho^2-1}\sum_{k=1}^{n}\frac{(2k-2)!!}{(2k-1)!!}\rho^{2(n-k)}-$$
$$-\begin{cases}4(P_0\beta\left\{E(\gamma\rho)-E\left(\arcsin\frac{1}{\rho},\ \gamma\rho\right)\right\}, & 1<\rho\le\frac{1}{\gamma}\\ 4P_0\rho\beta\gamma\left\{E\left(\frac{1}{\rho\gamma}\right)-E\left(\arcsin\gamma,\ \frac{1}{\rho\gamma}\right)-\right. & \\ \left.-\left[1-\left(\frac{1}{\rho\gamma}\right)^2\right]\left[K\left(\frac{1}{\rho\gamma}\right)-F\left(\arcsin\gamma,\ \frac{1}{\rho\gamma}\right)\right]\right\}, & \rho>\frac{1}{\gamma}\end{cases} \tag{2.13}$$

$$C=\frac{(2n)!!}{(2n-1)!!}(\beta\gamma)^{2n}-2\pi P_0\beta\sqrt{1-\gamma^2} \tag{2.14}$$

$$Q=\frac{(2n)!!}{(2n+1)!!}\frac{4n(\beta\gamma)^{2n+1}}{\pi}-P_0\beta^2(\pi-2\arcsin\gamma+2\gamma\sqrt{1-\gamma^2}) \tag{2.15}$$

where $F(x,\psi)$ and $E(x,\psi)$ are the incomplete elliptic integrals of the first and second kind.

When deriving relations (2.12)–(2.15) we used the value of the integrals [6, 9, 10]

$$\int_0^1\frac{y^{2n-1}}{\sqrt{1-y^2}}dy=\frac{(2n-2)!!}{(2n-1)!!}$$

$$\int_\rho^1\frac{x^{2n-1}}{\sqrt{x^2-\rho^2}}dx=\frac{(2n-2)!!}{(2n-1)!!}\sqrt{1-\rho^2}\sum_{k=1}^{n}\frac{(2k-3)!!}{(2k-2)!!}\rho^{2(n-k)}$$

$$\gamma\int_0^1\frac{\sqrt{(1/\gamma)^2-x^2}}{\sqrt{\rho^2-x^2}}dx=\begin{cases}E\left(\arcsin\frac{1}{\rho},\ \gamma\rho\right), & 1<\rho\le\frac{1}{\gamma}\\ \gamma\rho\left[E\left(\arcsin\gamma,\ \frac{1}{\gamma\rho}\right)-\left(1-\left(\frac{1}{\gamma\rho}\right)^2\right)F\left(\arcsin\gamma,\ \frac{1}{\gamma\rho}\right)\right], & \rho>\frac{1}{\gamma}\end{cases}$$

$$\int_0^1\rho^{2n-1}\sqrt{1-\rho^2}\,d\rho=\frac{(2n-2)!!}{(2n+1)!!} \tag{2.16}$$

and the sums

$$\sum_{m=0}^{\infty} \frac{(2m-1)!!}{(2m)!!} x^{2m} = \frac{1}{\sqrt{1-x^2}} \tag{2.17}$$

and also the formulae obtained by term-by-term integration of series of (2.17), and the value of the integral obtained using integration by parts

$$\int_0^1 \frac{x^{2m}dx}{\sqrt{\rho^2 - x^2}} = \frac{(2m-1)!!}{(2m)!!} \rho^{2m} \left[\arcsin \frac{1}{\rho} - \sqrt{\rho^2 - 1} \sum_{k=1}^{m} \frac{(2k-2)!!}{(2k-1)!!} \rho^{-2k} \right]$$

Expressions (2.12), (2.14) and (2.15) are identical with the results obtained previously in [10] when $P_0 = 0$.

To determine the remaining unknown parameters of the problem γ, β and P_0, we use relations (1.1), (1.2) and (1.6). We convert formula (1.1) using (2.13) and (2.14). We then obtain the following expression for the dimensionless value of the gap $H = A^{1/(2n-1)}$

$$H(\rho) = \left(\rho^{2n} - \frac{(2n)!!}{(2n-1)!!} (\beta\gamma)^{2n} - 2\pi P_0 \beta \sqrt{1-\gamma^2} \right) \left(1 - \frac{2}{\pi} \arcsin \frac{1}{\rho} \right) +$$

$$+ \frac{2\rho^{2n}}{\pi} \sqrt{\rho^2 - 1} \sum_{k=1}^{n} \frac{(2k-2)!!}{(2k-1)!!} \rho^{-2k} + 4P_0\beta \left[E(\rho\gamma) - E\left(\arcsin \frac{1}{\rho},\ \rho\gamma \right) \right] \tag{2.18}$$

Substituting (2.18) into (1.2) and (1.6) and using the values of the integrals [9]

$$\int_1^{1/\gamma} \rho E(\rho\gamma) d\rho = \frac{1}{3\gamma^2}(2 + \chi)$$

$$\int_1^{1/\gamma} \rho E\left(\arcsin \frac{1}{\rho},\ \rho\gamma \right) d\rho = \frac{1}{3\gamma^2}(3\gamma - \gamma^3 + \chi)$$

where

$$\chi = -(1+\gamma^2)E(\gamma) + (1-\gamma^2)K(\gamma)$$

and also formula (2.16) and the value of the integral obtained by integration by parts

$$\int_1^{1/\gamma} \rho^{2n+1} \arcsin \frac{1}{\rho} d\rho = \frac{1}{(2n+2)\gamma^{2n+2}} \left[\arcsin \gamma - \frac{\pi}{2} \gamma^{2n+2} + \right.$$

$$\left. + \gamma \sqrt{1-\gamma^2} \frac{(2n)!!}{(2n+1)!!} \sum_{k=0}^{n} \frac{(2k-1)!!}{(2k)!!} \gamma^{2(n-k)} \right]$$

we obtain a quadratic equation in P_0, solving which we obtain

$$P_0 = \frac{B_2 - \sqrt{B_2^2 - 4B_1 K}}{2B_1} \tag{2.19}$$

$$B_1 = 2\beta(1 - \gamma + \varphi\sqrt{1-\gamma^2})$$

$$B_2 = \frac{\beta^{2n}}{\pi} \left\{ \left(\frac{(2n)!!}{(2n-1)!!} \gamma^{2n} - 1 \right) \varphi + \sqrt{1-\gamma^2} \sum_{k=1}^{n} \frac{(2k-2)!!}{(2k-1)!!} \gamma^{2k-1} \right\}$$

$$\varphi = \arcsin \gamma - \pi/2$$

(the minus sign in front of the radical is chosen because when $K = 0$ the equation $P_0 = 0$ must be satisfied), and we will also have the equation

$$V = 2\beta^{2n+2}\left\{\frac{(2n)!!(2n-1)}{(2n+1)!!}\gamma^{2n+2}\sqrt{1-\gamma^2} + \frac{\sqrt{1-\gamma^2}}{n+1}\sum_{k=0}^{n}\frac{(2k)!!}{(2k+1)!!}\gamma^{2k+1} + \right.$$

$$\left. + \left(\frac{(2n)!!}{(2n-1)!!}\gamma^{2n} - \frac{1}{n+1}\right)\varphi\right\} - \frac{4\pi}{3}P_0\beta^3(4 - 3\gamma - \gamma^3 + 3\varphi\sqrt{1-\gamma^2}) \tag{2.20}$$

Substituting (2.19) into (2.20), we obtain an equation containing the unknown quantities β and γ. This equation was solved numerically for β for given γ, and then, using (2.19), we determined the dimensionless pressure in the liquid P_0. After this, the remaining characteristics of the problem were found from (2.12)–(2.15). Expression (2.15) serves to determine the load Q corresponding to the chosen value of γ. If we are given the value of the load Q, the unknown quantities β and γ can be found by solving the system of equations (2.2) and (2.15).

If we neglect the elastic deformations of the half-space outside the contact area, i.e. we assume that $|u(r) - u(a)| \ll |f(r) - f(a)|$ for $r \geqslant a$, conditions (1.2) and (1.6) can be reduced to the simple form

$$P_0 = \frac{2K}{\beta^{2n}(1-\gamma^{2n})} \tag{2.21}$$

$$\beta = \left[\frac{V(n+1)}{\pi(1-(n+1)\gamma^{2n} + n\gamma^{2n+2})}\right]^{1/(2n+2)} \tag{2.22}$$

Relations (2.21) and (2.22), together with (2.12)–(2.15) give the analytical solution of the problem in parametric form.

3. RESULTS OF CALCULATIONS

We investigated the solution of the problem as a function of the values of the following parameters: V, determined by the volume of liquid in the meniscus and the geometry of the punch, Q, representing the load applied to the punch, and K, which depends on the surface tension of the liquid and the elastic properties of the half-space. Here we determined the dimensionless functions of the contact pressure $P(\rho)$ and the displacements of the boundary of the elastic half-space $U(\rho)$, and also the dimensionless values of the pressure in the liquid P_0, the radius of the contact area α, the width $(\beta - \alpha)$ of the ring-shaped region occupied by the liquid, and the indentation C.

In addition to the above quantities, we introduce into consideration the capillary adhesion force

$$f_a = q_0 - q \tag{3.1}$$

where q and q_0 are the loads required to form the contact area of radius a when there is no meniscus and when there is a meniscus, respectively. From (2.15) and (3.1), we obtain the following equation for the dimensionless capillary adhesion force

$$F_a = A^{2/(2n-1)}\frac{f_a}{p^*} = P_0\beta^2(\pi - 2\arcsin\gamma + 2\gamma\sqrt{1-\gamma^2}) \tag{3.2}$$

Note that, if we put $n = 1$ and $A = 1/(2R)$ in (3.2), and also substitute P_0 as given by the simplified formula (2.21), we can obtain the following simplified expression for the adhesion force for small γ

$$f_a \approx 4\pi R\sigma(1 + (a/b)^2)$$

similar to formula (0.1), obtained previously [5].

In Fig. 2 we show graphs of the dimensionless pressure at the contact for two forms of punch, corresponding to $n = 1$ (curves 1) and $n = 2$ (curves 2), with $Q = 0$, $K = 10^{-4}$ and $V = 10^{-4}$ (the continuous curves). The dashed curves correspond to the pressure distributions for the same values of the radius of the contact area α when there is no liquid. In these graphs the values of δ define the external radius of the ring-shaped region occupied by the liquid, referred to the radius of the contact area, i.e. $\delta = b/a = 1/\gamma$. A comparison of the curves indicates that for the same contact area the pressure under the punch when a meniscus is present is less than in the case of a dry contact. Hence, in particular, it follows that the value of the adhesion force, introduced by (3.1), will be positive. At the edges of the contact area the pressures become negative (for curve 2 we have $P_0 = 2.05$). This means that the pressure acting on certain parts is less than the atmospheric pressure. The shape of the punch has a considerable effect on the pressure distribution diagram and also on the width of the ring-shaped region occupied by the liquid.

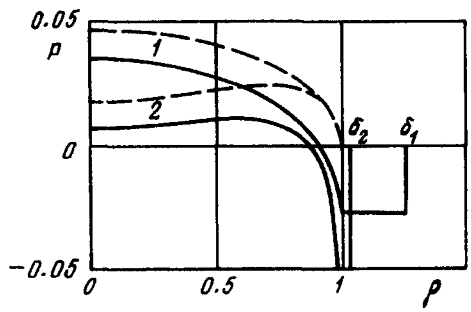

Fig. 2.

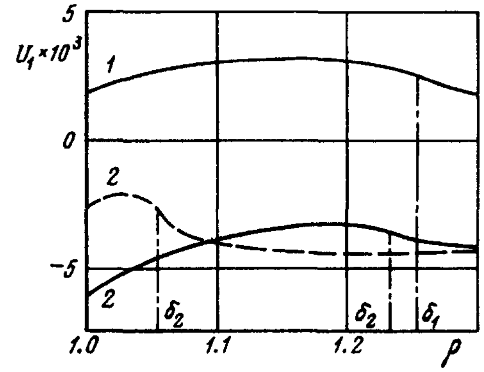

Fig. 3.

Graphs of the function $U_1(\rho) = -U(\rho)$, illustrating the form of the elastic half-space outside the contact area, are shown in Fig. 3 for $n = 1$, $K = 10^{-4}$ and $Q = 0$ (curve 1) and $Q = 10^{-3}$ (curves 2), for $V = 10^{-4}$ (the continuous curves) and $V = 2 \times 10^{-4}$ (the dashed curve). The results show that, when a meniscus is present, the boundary of the elastic half-space is considerably distorted and there is a discontinuity in the derivative of the shape of the surface on the external circle of radius δ_i of the ring-shaped region occupied by the liquid.

In Fig. 4 we show graphs of the radius of the contact area α, the width of the ring-shaped region occupied by the liquid $(\beta - \alpha)$, and the indentation C as a function of the load Q for $n = 1$ with $K = 2 \times 10^{-4}$ (curves 1) and $K = 10^{-4}$ (curves 2). The continuous curves correspond to $V = 10^{-4}$ and the dashed curves correspond to $V = 2 \times 10^{-4}$. We can conclude from these curves that the contact area and the indentation of the punch are non-zero for certain negative loads and exceed the corresponding Hertz values (the dash–dot curves) for positive loads. This effect is more appreciable the greater the value of K. An increase in the volume of the liquid leads to a reduction in the contact area and in the indentation, and the ring of liquid becomes wider. A characteristic feature of the data, and also of the graphs in Figs 5 and 6, is the non-uniqueness of the contact characteristics in a certain region of negative values of the load.

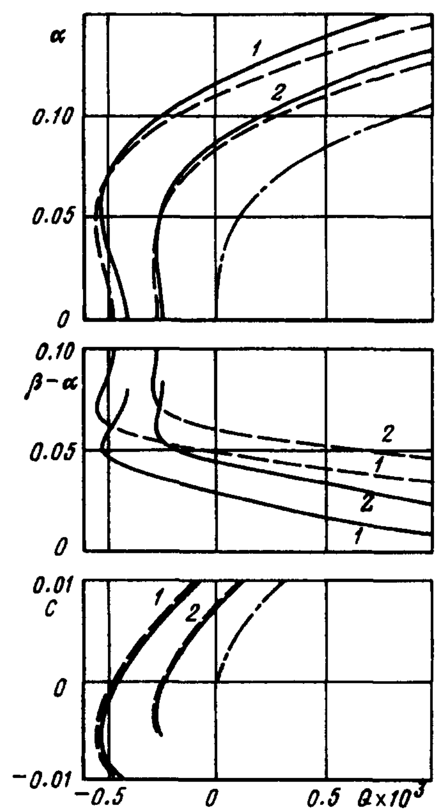

Fig. 4.

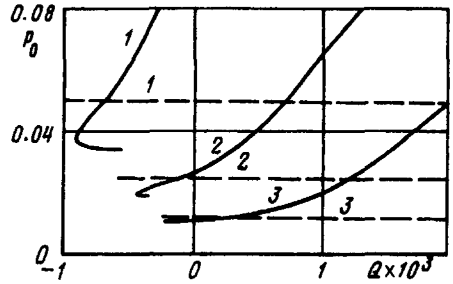

Fig. 5.

In Fig. 5 we show graphs of the dimensionless pressure in the liquid P_0 as a function of the dimensionless load Q for $n = 1$, $V = 10^{-4}$ and $K = 2\times10^{-4}$ (curves 1), $K = 10^{-4}$ (curves 2) and $K = 5\times10^{-5}$ (curves 3). The continuous curves represent the results obtained using the exact formulae (2.19) and (2.20), while the dashed curves are those obtained using the simplified formulae (2.21) and (2.22). It can be seen that, using the simplified approach, the pressure in the liquid is found to be independent of the load, whereas calculations using the exact relations show that, beginning at a certain value of Q, P_0 increases as the load increases. This disagreement is greater than the greater the load Q and the greater the value of K.

Note that when $n > 1$, calculations using the simplified relations (2.21) and (2.22) show that P_0 decreases as Q increases. In the results obtained using relations (2.19) and (2.20), this reduction only occurs for fairly small values of Q, after which P_0 increases as the load increases.

Figure 6 illustrates the dimensionless adhesion force F_a, calculated from (3.2), as a function of the load for $n = 1$ (Fig. 6a) and $n = 3$ (Fig. 6b). The values of the parameters for which these graphs were drawn are shown in Table 1. The continuous curves represent the results of calculations using the exact formulae (2.19) and (2.20), while the dashed curves represent the results obtained using the simplified formulae (2.21) and (2.22).

In these graphs the initial points of all the curves lie on one straight line. In fact, it follows from (2.15) and (3.2) that when $\gamma = 0$ we have $F_a = -Q$. The results also show that the adhesion force increases as the load increases, beginning from a certain value of Q, and this increase is sharper the smaller the volume of the liquid in the meniscus and the more mildly sloping the shape of the punch. In this case the adhesion force is greater the larger the values of K.

An analysis of the results shown in Fig. 6 enables us to conclude that calculations carried out using the simplified relations (2.21) and (2.22) give a considerable error in determining the adhesion force, particularly for large values of the load Q. The least disagreement between the results occurs when $n = 1$ and for values of Q that are small in absolute value. When $n > 1$, the results differ not only quantitatively but also qualitatively.

4. CONCLUSIONS

An analysis of the results of the solution of the problem of the indentation of an axisymmetric punch into an elastic half-space when there is a liquid forming a meniscus in the gap enables us to draw the following conclusions.

1. Capillary forces have a considerable effect on the contact characteristics when there is interaction between elastic solids. In particular, the presence of a meniscus leads to the occurrence under the punch of pressures less than atmospheric.

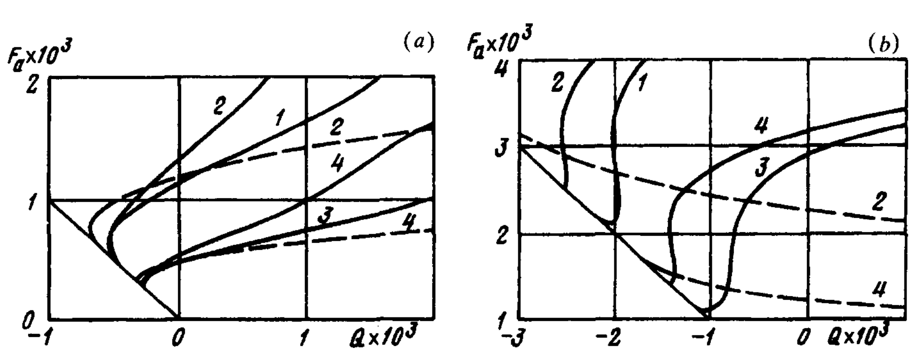

Fig. 6.

Table 1

K	V: 2×10^{-4} (a), 2×10^{-3} (b)	V: 10^{-4} (a), 10^{-4} (b)
5×10^{-4} (a), 2×10^{-5} (b)	1	2
10^{-4} (a), 10^{-5} (b)	3	4

2. The influence of capillary effects is stronger the smaller the amount of liquid in the gap and the more mildly sloping the shape of the interacting surfaces.

3. An adhesion force occurs between the contacting solids, which increases as the load increases. The action of this force increases the contact area and also gives rise to a non-unique relationship between the radius of the contact area and the other contact characteristics and the load over a certain range of negative values of the loads. Qualitatively similar results were obtained when investigating adhesion in a dry contact in [7, 11], which indicates the similarity between the occurrence of adhesion when there is contact between lubricated and dry surfaces.

4. The analysis has enabled us to determine the limits of applicability of the simplified approach, which enables analytic solutions of the problem to be obtained assuming that the elastic deformations of the surfaces outside the contact area are negligibly small. In particular, we have shown that for a parabolic-shaped punch the simplified approach can be used for loads that are small in absolute value. For punch shapes described by higher-order polynomials, the contact characteristics must be calculated using the exact formulae.

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